Probability Theory for Financial Applications

The Pre-MFE Program at Baruch College

February 3 – March 31, 2022

All the Pre-MFE seminars in the Spring 2022 semester will be offered online, via Zoom. Registration is limited to 40 students.

The seminar addresses fundamental concepts of (calculus based) undergraduate probability that are most relevant for a smooth transition to Baruch MFE program. Special attention will be paid to (1) probabilistic ideas, language, and notation; (2) examples and models that are relevant to financial engineering.

Probabilistic topics (selected):

- Discrete distributions.
- Continuous distributions.
- Computation of expectations, variances, and generating functions.
- Joint distributions.
- Conditional distributions and conditional expectations.
- Laws of large numbers.
- Central limit theorem.

Financial topics (selected):

- Binomial asset pricing model.
- Risk and expected return of a portfolio.
- Black-Scholes model.
- Risk-neutral probabilities.
- Options pricing.
- Monte-Carlo simulation.

Dates and Times:
Dates: February 3, 10, 17, 24, March 3, 10, 17, 24
Times: Lectures 7-10pm, Review Session 10-11pm, New York time
Final Exam: March 31, 7-9pm, New York time

Instructor: Elena Kosygina, Faculty, Baruch College Financial Engineering Program

Tuition: $1,450

Certification: Upon successfully completing the Probability Theory for Financial Applications seminar and passing the final exam, a Certificate of Completion will be issued by the Baruch MFE Program. A Certificate of Completion with Distinction will be issued to every participant completing the seminar with an average above 90%.

Attending the Probability Theory for Financial Applications and passing the final exam meets the probability pre-requisite for the Baruch MFE Program. Upon request, recommendation letters reflecting performance in the seminar will also be provided.

Registration: To register or to receive more information about the Pre-MFE Probability Seminar, send an email to baruch.mfe@baruch.cuny.edu

Textbooks:

- A Natural Introduction to Probability Theory by Ronald Meester, Birkhauser, 2nd Edition, ISBN 978-3-7643-8723-5.
- Instructor's notes (posted on the course web page for every session)

Prerequisites: Multivariable calculus and some previous exposure to probability (for example, a probability or statistics course previously taken).

Students should read in advance the following sections from the textbook:
Chapter 1, Sections 1.1–1.3 and do all exercises within the text for these sections and exercises 1.7.1–1.7.3 from Section 1.7.

Detailed Syllabus

Session 1:

- Random experiments. Events and operations with them.
- Counting and combinatorics.
- Probability measure and its properties.
- Conditional probabilities.
- Independence of events.

Financial applications:

- Binomial asset pricing model.
- Market probabilities versus risk-neutral probabilities.

Textbook sections: Chapter 1 and instructor's notes.

Session 2:

- Discrete random variables. Probability mass function. Distribution function.
- Independence of random variables.
- Expectation. Variance.

Financial applications:

- Most frequently used discrete distributions: binomial, Poisson, geometric, negative binomial, hypergeometric (time permitting).
- Modeling with discrete distributions: which one to use for a given set of data?

Textbook sections: Sections 2.1–2.3 and instructor's notes.

Session 3:

- Random vectors (discrete case).
- Covariance and correlation.
- Conditional distributions and expectations (discrete case).
- Moment generating function (time permitting).

Financial applications:

- Risk and expected return of a portfolio.
- Calibration of a binomial model.
- Pricing of European derivative securities (binomial model).

Textbook sections: Sections 2.4–2.6 and instructor's notes.

Session 4:

- Random walk on integers. Path counting.

- First passage times.
- Reflection principle.

Financial applications:

- Pricing of path-dependent options.
- An idea of dynamic programming: pricing of American options.

Textbook sections: Chapter 3 and instructor's notes.

Session 5:

- Probability density functions. Continuous random variables.
- Expectation.
- Random vectors and independence.

Financial applications:

- Most frequently used continuous distributions: uniform, exponential, normal, gamma, lognormal.
- Geometric Brownian motion and Black-Scholes model.

Textbook sections: Sections 5.1–5.6 and instructor's notes.

Session 6:

- Functions of random variables and vectors.
- More about expectation. Variance. Covariance and correlation.
- Conditional distributions and expectations.

Financial applications:

- Distributions of the sum, minimum, and maximum of several random variables. Order statistics (time permitting).
- What are "fat tails" and where do they appear in practice?

Textbook sections: Sections 5.7–5.10 and instructor's notes.

Session 7:

- Infinitely many repetitions. Sequences of i.i.d. random variables.
- Laws of large numbers.
- Central limit theorem.

Financial applications:

- Monte-Carlo simulation: parameter and probability estimation, numerical integration.
- Black-Scholes model as a limit of binomial models.

Textbook sections: Chapter 4, Section 5.11, Sections 6.1–6.4, and instructor's notes.

Session 8:

- Simulation methods: inverse transformation method, acceptance-rejection method.
- Variance reduction techniques (time permitting).

Financial applications:

- Sampling from frequently used distributions.
- Monte-Carlo simulation: pricing of path-dependent options.

Textbook sections: Instructor's notes.