Advanced Calculus with Financial Engineering Applications

The Pre-MFE Program at Baruch College

October 14 - December 16, 2019

Mathematical and financial concepts that are fundamental for a successful learning experience in financial engineering graduate programs will be presented and explained in detail. Strong emphasis will be placed on fully understanding the material.

Mathematical topics (selected):

- Numerical integration methods
- Lagrange multipliers
- Convergence of Taylor series expansions
- Finite difference approximations
- Stirling's formula
- Polar coordinates transformations
- Newton's method for higher dimensional problems

Financial topics (selected):

- Bond duration and convexity
- Dollar Duration and DV01; bond portfolio hedging
- Put–Call parity
- Black–Scholes formula
- Δ–hedging and Γ–hedging
- Numerical estimation of the Greeks
- Implied volatility
- Bootstrapping for finding interest rate curves

Dates and Times:
Lectures: October 14, 21, 28, November 4 or 11, 18, 25, December 2, 9, 6-10pm
Final Exam: December 16, 6-9pm

Instructor: Rados Radoicic, Professor, Baruch College Financial Engineering Program

Tuition: $1,450

Certification: Upon successfully completing the Advanced Calculus with Financial Engineering Applications and passing the final exam, a Certificate of Completion will be issued by the Baruch MFE Program. A Certificate of Completion with Distinction will be issued to every participant completing the seminar with an average above 90%.

Attending the seminar on Advanced Calculus with Financial Engineering Applications and passing the final exam meets the calculus prerequisites for the Baruch MFE Program. Upon request, recommendation letters reflecting performance in the seminar will also be provided.

Registration: To register or to receive more information about the Advanced Calculus with Financial Applications Seminar, send an email to **baruch.mfe@baruch.cuny.edu**

Textbooks:

"A Primer for the Mathematics of Financial Engineering", Second Edition, by Dan Stefanica, FE Press, 2011.
"Solutions Manual – A Primer for the Mathematics of Financial Engineering", Second Edition, by Dan Stefanica, FE Press, 2011.

Prerequisites: Students should read in advance the following sections from the textbook:
Chapter 1, Sections 1.1 - 1.10
Chapter 2, Sections 2.6, 2.7
Chapter 3, Sections 3.1 - 3.4
and do the exercises from the first chapter (Section 1.12).

Detailed Syllabus

Session 1:

- Brief review of elementary calculus from a more formal point of view: product rule, quotient rule, chain rule.
- Antiderivatives and definite integrals. Fundamental Theorem of Calculus. Integration by substitution and connection to chain rule. Integration by parts and connection to Product Rule.
- Differentiating definite integrals with respect to the integral limits and with respect to a parameter under the integral sign.
- Partial Derivatives.
- The gradient and the Hessian of multivariable functions.

Financial Applications:

- European Call and Put options. Payoff and P&L at maturity. American Call and Put options.
- Options portfolios: bull spread, bear spread, butterfly spread, straddle, strangle, collar, risk reversal. Payoff and P&L at maturity.
- Replicating piecewise linear payoffs with option portfolios.
- The concept of no–arbitrage. The Generalized Law of One Price.
- Put–Call parity.
- Arbitraging the Put–Call parity. Arbitraging the Put–Call parity with Bid–Ask spreads.

Textbook Sections: Chapter 1.

Session 2:

- Convergence and evaluation of improper integrals.
- Differentiating improper integrals with respect to the integral limits.
- Numerical methods for approximating definite integrals: the Midpoint, Trapezoidal, and Simpson's rules.
- Convergence of numerical algorithms – practical considerations. Approximation errors and the order of convergence of a numerical algorithm.
- The order of convergence of the midpoint, trapezoidal, and Simpson's rules.

Financial Applications:

- Discount factors and discount curve.
- Discretely compounded interest and continuously compounded interest.
- Interest rate curves. Zero Rates. Forward rates and instantaneous forward rates.
- Bond Pricing. Yield of a Bond. Bond Duration. Convexity of a bond.
- Bond pricing and bond yield with discretely compounded interest.
- Numerical implementation of bond mathematics.

Textbook Sections: Chapter 2.

Session 3:

- Discrete probability, mean and variance.
- Random variables. Density function and cumulative distribution. Mean and Variance for random variables.
- The standard normal variable.
- Normal random variables. Mean and Variance.

Financial Applications:

- A lognormal model for the evolution of stock prices.
- The Black–Scholes formula.
- Computing the Black–Scholes formula using numerical integration methods.
- Numerical implementation and accuracy of the Black–Scholes formula.
- Formulas for the Greeks of plain vanilla European Call and Put options. The magic of Greek computations.
- The concept of hedging. Δ–hedging and Γ–hedging for single options and for options portfolios.
- Numerical computations of the Greeks.

Textbook Sections: Chapter 3.

Session 4:

- Change of density function for functions of random variables.
- Lognormal random variables.
- Connection between the density functions of normal and lognormal variables.
- Independent random variables.
- Sums of independent normal random variables. Products of independent lognormal random variables.

Financial Applications:

- Implied volatility. Put–Call parity and consequences for implied volatility.
- Computing implied volatility from the Black–Scholes model.
- Risk–neutral pricing and the Black–Scholes formula.
- Interpretation of the $N(d_1)$ and $N(d_2)$ terms from the Black–Scholes formula.
- Actual and risk–neutral probabilities that European call or put options expire in the money.

Textbook Sections: Chapter 4.

Session 5:

- Newton's method, bisection method, and secant method for solving nonlinear equations.
- Taylor's approximations: infinite series and integral residual. Convergence issues. Taylor's approximations for multivariable functions.

Financial Applications:

- Numerical computation of bond yields.
- Bootstrap method for finding the zero-rate curve.
- Approximation of the Black–Scholes formula for at–the–money options.

Textbook Sections: Chapters 5 and 6.

Session 6:

- Finite Difference approximations for first order derivatives: forward, backward, central approximations.
- Finite Difference approximations for higher order derivatives. Order of approximation.

- Convergence of infinite series. Radius of convergence. Stirling's formula.
- Taylor series expansions. Taylor expansions for exponential and logarithmic functions.

Financial Applications:

- Computing the Greeks of derivative securities without closed formulas: finite difference approximations.
- Percentage and log returns.
- Changes in bond yields for small parallel shifts in the zero rate curve.
- Dollar duration and dollar convexity for bonds and bond portfolios. DV01.
- Approximating bond portfolio returns using DV01 and dollar convexity.
- Bond portfolio immunization.
- Connections between duration, convexity, and the relative change in the value of a bond for parallel shifts in the yield curve.

Textbook Sections: Chapter 7.

Session 7:

- Chain rule for multivariable functions.
- Double integrals. Switching the order of integration (Fubini's Theorem).
- Change of Variables for Double Integrals. Polar coordinates.
- Finding relative extrema for functions of several variables.
- The Black–Scholes PDE.
- Change of variables for reducing the Black–Scholes PDE to the heat PDE.

Financial Applications:

- Show that the density function of the standard normal distribution has unit integral over $\mathbb{R}$.
- Financial interpretation of the terms from the Black–Scholes PDE.
- The early exercise of American options on non–dividend–paying assets.
- The Greeks and the Black–Scholes PDE.
- When do Θ and Γ have different signs?
- Barrier Options. Arbitrage arguments for the up-and-in up-and-out parity.

Textbook Sections: Chapter 8.

Session 8:

- Lagrange multipliers for finding absolute extrema of multivariable functions.
- N–dimensional Newton's method.

Financial Applications:

- Optimal investment portfolios.
- Minimum variance portfolios. Maximum return portfolios.

Textbook Sections: Chapter 9.