Probability Theory for Financial Applications

The Pre-MFE Program at Baruch College

October 19 - December 21, 2017

The seminar addresses fundamental concepts of (calculus based) undergraduate probability that are most relevant for a smooth transition to Baruch MFE program. Special attention will be paid to (1) probabilistic ideas, language, and notation; (2) examples and models that are relevant to financial engineering.

Probabilistic topics (selected):

- Discrete distributions.
- Continuous distributions.
- Computation of expectations, variances, and generating functions.
- Joint distributions.
- Conditional distributions and conditional expectations.
- Laws of large numbers.
- Central limit theorem.

Financial topics (selected):

- Binomial asset pricing model.
- Risk and expected return of a portfolio.
- Black-Scholes model.
- Risk-neutral probabilities.
- Options pricing.
- Monte-Carlo simulation.

Dates and Times:
Lectures: October 19, 26, November 2, 9, 16, 30, December 7, 14, 6-10pm
Final Exam: December 21, 6-9pm

Instructor: Ivan Matic, Faculty, Baruch College Financial Engineering Program

Tuition: $1,450

Attending the Probability Theory for Financial Applications and passing the final exam meets the probability pre-requisite for the Baruch MFE Program. Upon request, recommendation letters reflecting performance in the seminar will also be provided.

Registration: To register or to receive more information about the Pre-MFE Probability Seminar, send an email to **baruch.mfe@baruch.cuny.edu**

Textbooks:

• *A Natural Introduction to Probability Theory* by Ronald Meester, Birkhauser, 2nd Edition, ISBN 978-3-7643-8723-5.

• Instructor's notes (posted on the course web page for every session)

Prerequisites: Multivariable calculus and some previous exposure to probability (for example, a probability or statistics course previously taken).

Students should read in advance the following sections from the textbook:
Chapter 1, Sections 1.1–1.3 and do all exercises within the text for these sections and exercises 1.7.1–1.7.3 from Section 1.7.

Detailed Syllabus

Session 1:

- Random experiments. Events and operations with them.
- Counting and combinatorics.
- Probability measure and its properties.
- Conditional probabilities.
- Independence of events.

Financial applications:

- Binomial asset pricing model.
- Market probabilities versus risk-neutral probabilities.

Textbook sections: Chapter 1 and instructor's notes.

Session 2:

- Discrete random variables. Probability mass function. Distribution function.
- Independence of random variables.
- Expectation. Variance.

Financial applications:

- Most frequently used discrete distributions: binomial, Poisson, geometric, negative binomial, hypergeometric (time permitting).
- Modeling with discrete distributions: which one to use for a given set of data?

Textbook sections: Sections 2.1–2.3 and instructor's notes.

Session 3:

- Random vectors (discrete case).
- Covariance and correlation.
- Conditional distributions and expectations (discrete case).
- Moment generating function (time permitting).

Financial applications:

- Risk and expected return of a portfolio.
- Calibration of a binomial model.
- Pricing of European derivative securities (binomial model).

Textbook sections: Sections 2.4–2.6 and instructor's notes.

Session 4:

- Random walk on integers. Path counting.
- First passage times.
- Reflection principle.

Financial applications:

- Pricing of path-dependent options.
- An idea of dynamic programming: pricing of American options.

Textbook sections: Chapter 3 and instructor's notes.

Session 5:

- Probability density functions. Continuous random variables.
- Expectation.
- Random vectors and independence.

Financial applications:

- Most frequently used continuous distributions: uniform, exponential, normal, gamma, log-normal.
- Geometric Brownian motion and Black-Scholes model.

Textbook sections: Sections 5.1–5.6 and instructor's notes.

Session 6:

- Functions of random variables and vectors.
- More about expectation. Variance. Covariance and correlation.
- Conditional distributions and expectations.

Financial applications:

- Distributions of the sum, minimum, and maximum of several random variables. Order statistics (time permitting).
- What are "fat tails" and where do they appear in practice?

Textbook sections: Sections 5.7–5.10 and instructor's notes.

Session 7:

- Infinitely many repetitions. Sequences of i.i.d. random variables.
- Laws of large numbers.
- Central limit theorem.

Financial applications:

- Monte-Carlo simulation: parameter and probability estimation, numerical integration.
- Black-Scholes model as a limit of binomial models.

Textbook sections: Chapter 4, Section 5.11, Sections 6.1–6.4, and instructor's notes.

Session 8:

- Simulation methods: inverse transformation method, acceptance-rejection method.
- Variance reduction techniques (time permitting).

Financial applications:

- Sampling from frequently used distributions.
- Monte-Carlo simulation: pricing of path-dependent options.

Textbook sections: Instructor's notes.