

MTH 9862 Assignment 10 (4/30/2010 - 5/6/2010).

- (1) Assume the BSM model. Define

$$Y(T) = \exp\left(\frac{1}{T} \int_0^T \log S(t) dt\right).$$

Suppose that an Asian option has a payoff equal to $(Y(T) - K)_+$ at time T . Find an explicit formula for the price of this option at time 0. Use Jensen's inequality to compare the price of this option with the price of the Asian option with the same expiration T and the payoff

$$\left(\frac{1}{T} \int_0^T S(t) dt - K\right)_+.$$

Which one is larger?

- (2) Calculate the time 0 price of an Asian option, with strike price K , in which the average of the stock price is calculated on the basis of just three sampling times 0, $T/2$, and T , where T is the maturity of the contract.
- (3) 7.8.
- (4) Assume the BSM model ($r \neq 0$). Consider the floating strike Asian call option with payoff at time T given by

$$\left(\frac{1}{c} \int_{T-c}^T S(t) dt - S(T)\right)_+.$$

Follow the methodology described in lecture notes or in Shreve II, pp. 324-329 to derive an analog of Theorem 2 (Theorem 7.5.3 in Shreve II) for this option. (Hint: the game here is to find explicitly the process $\gamma(t)$ and the portfolio process $X(t)$. The rest, as you will see, is just the repetition of the calculations done in the lecture.)

- (5) Extra credit: (5 points added to the sum of your quiz grades; hand in to me individually together with your quiz on May 6). Let $\rho(t)$ be a given probability density function on $[0, T]$. Assume the BSM model ($r \neq 0$). Consider an Asian option with maturity T and payoff

$$\left(\int_0^T S(t)\rho(t) dt - K\right)_+.$$

Follow the methodology described in lecture notes or in Shreve II, pp. 324-329 to derive an analog of Theorem 2 (Theorem 7.5.3 in Shreve II) for this option.