MTH 9862 Assignment 1 (1/29/2010 - 2/04/2010).

(1) Let (X_1, X_2) have the following joint moment generating function (λ, μ, ν are positive parameters)

$$M_{(X_1,X_2)}(t_1,t_2) = \exp\left(\lambda(e^{t_1} - 1) + \mu(e^{t_2} - 1) + \nu(e^{t_1+t_2} - 1)\right).$$

Find the correlation between X_1 and X_2.

(2) Compute the moment generating function of a normal random variable with mean μ and variance σ^2.

(3) Read pp. 340-342 of S. Shreve, vol. 2 concentrating on the Example 8.2.3. Compare with the discrete case.

(4) Let $(B(t))_{t\geq 0}$ be a Brownian motion and $a, b > 0$. Define

$$\tau_{-a} = \inf\{t \geq 0 : B(t) = -a\}, \quad \tau_b = \inf\{t \geq 0 : B(t) = a\}, \quad \tau = \tau_{-a} \wedge \tau_b.$$

(i) Prove that $P(\tau_{-a} < \tau_b) = b/(a + b)$.

(ii) Compute $E\tau$.