

MTH 9814  
Fall 2010  
Homework 1

Hull:

Problems 4.1, 4.4, 4.5, 4.6, 4.9, 4.14, 4.24, 4.25

Programming Exercises:

Excel / VBA:

(1) Construct the system of equations satisfied by the coefficients of a cubic spline for the following interest rates:

t	zero rate
0	0.04
1	0.042
2	0.043
5	0.045

Then, in a worksheet, use the array functions MINVERSE and MMULT to solve the linear system of equations and determine the spline coefficients.

- Submit the spline coefficients resulting from this step.

(2) Write a user-defined function (UDF) in VBA that takes a Range formatted like the above as an argument and returns the spline coefficients. Check it against the worksheet results from (1) above.

- Submit the text of the UDF from this step.

(3) Write a UDF that takes two Ranges as arguments—one containing a time, a second containing spline coefficients formatted like the output from your UDF in (2) above—and returns the zero rate to that time by performing the spline interpolation. Test it on the full range of times running from 0 to 5, spaced 0.05 apart. In a worksheet, graph the result.

- Submit the text of the UDF from this step, and the resulting graph.

(4) Using your UDF's, produce the graph as above in (3) that results when the 1-year zero rate is shocked upward by 10 basis points, but the other rates remain the same. (1bp is 1% of 1%, or 0.0001.) Compare the two graphs. What do you notice about the effect of the shock? Is it what you would expect? Does the result make financial sense?

- Submit the graph from this step and the answer to this discussion question.

C++:

(1) Implement a class to represent discount factor curves. Give it the ability to calculate the following:

(a) The discount factor at any time in the curve's domain using either linear spot rate interpolation or constant forward rate interpolation.

(b) The zero rate at any time as above in (a), quoted either in terms of simple interest, discretely compounded interest, or continuously compounded interest.

(c) The forward rate between any two times in the curve's domain, quoted in any of the terms described above in (b).

To test your code, use the rates given above and calculate the 1-month forward rates at times from 0 to 59 months, spaced no more than 1 month apart. Do this using both of the interpolation methods you implemented. How do the forward rates compare? Does one set of forward rates seem preferable to the other?

- Submit the header file that defines your discount factor class and a graph of your 1-month forward rates by the two methods described.

(2) Implement a class to represent a zero-coupon bond. Give it the ability to price when given a discount factor curve in the form of an object as created above in (1). Design your class with the idea in mind that you will later implement classes for other types of bonds.

- Submit the header file that defines your zero-coupon bond class.

Note that header files should be *appropriately commented* so that the grader can discern the use of each function. Imagine that your grader is a fellow programmer who will be using your code, and make sure it is clear what each function does and how it is used. Also, make sure that your comments describe any limitations or restrictions on the function arguments.