

MTH 9831 Real Analysis and Probability

Syllabus, Fall 2009

- *First examples of stochastic processes and an informal introduction of basic notions and tools.*
- 1. Random walks. Basic properties. Gambler's ruin problem. Informal introduction of the reflection principle and stopping times.
- 2. Pricing by arbitrage. The binomial asset pricing model.
- 3. Real-world and risk-neutral probabilities. Associated martingales. Change of measure in the discrete setting. Informal introduction of the geometric Brownian motion as a limit of binomial models.
- 4. The simplest continuous time processes: Poisson process and its variations.
 - *Measure-theoretic language and essential background.*
- 5. Sample space, sigma-algebras, probability measures. Random variables and their distributions.
- 6. Expectations. Basic properties. Change of variable formula. Integral convergence theorems. Change of measure. Radon-Nikodym theorem.
- 7. Joint distributions. Independence. Copulas.
- 8. Characteristic and moment generating functions. Gaussian random variables.
- 9. Weak convergence. Sums of independent random variables. Laws of large numbers and the central limit theorem. Product spaces. Fubini's theorem.
- 10. Different types of convergence. L^p spaces. Hölder and Minkowski inequalities.
- 11. Conditional expectations and conditional probability distributions.
- 12. Martingales. Stopping times. Optional stopping theorem.
 - *Brownian motion.*
- 13. Brownian motion: equivalent definitions, construction, basic properties. Reflection principle. The joint distribution of the Brownian motion and its running maximum.
- 14. Martingales associated with Brownian motion. Lévy's characterization of Brownian motion. Heat equation and Brownian motion semi-group. Geometric Brownian motion.