

# Numerical Linear Algebra for Financial Engineering

## The Pre-MFE Program at Baruch College

February 2 - March 30, 2017

A numerical view of linear algebra concepts that are fundamental for a successful learning experience in financial engineering graduate programs will be presented in this seminar. Emphasis will be placed on numerical linear algebra methods and their implementation, and on financial applications.

Mathematical topics (selected):

- Direct methods for solving linear systems. Backward and Forward Substitution. LU decomposition.
- Eigenvalues and eigenvectors. Diagonal decomposition.
- Eigenvalue methods. Power Method. QR Algorithm.
- Symmetric matrices. Diagonal Decomposition of symmetric matrices.
- Symmetric positive definite matrices. Diagonal dominance. Gershgorin's theorem.
- Multivariate normal random variables. Linear combinations of normal random variables.

Financial topics (selected):

- Efficient cubic spline interpolation. Bonds and swaps pricing.
- Arrow-Debreu Model. Risk neutral pricing.
- Complete and incomplete markets - binomial and trinomial trees.
- Covariance and correlation matrices from time series data.
- Linear Transformation Property.
- Generating normal samples with given correlation matrix.
- Least Squares. Linear Regression.
- Portfolio optimization theory. Efficient frontier and tangency portfolio.
- Value-at-Risk (VaR). Portfolios with multivariate normal asset returns. Portfolio VaR.

### Dates and Times:

Lectures: February 2, 9, 16, 23, March 2, 9, 16, 23, 2016, 6-10pm; Final Exam: March 30, 6-9pm

**Instructor:** Dan Stefanica, Director, Baruch College Financial Engineering Program

**Tuition:** \$1,450

**Certification:** Upon successfully completing the Advanced Calculus with Financial Engineering Applications and passing the final exam, a Certificate of Completion will be issued by the Baruch MFE Program. A Certificate of Completion with Distinction will be issued to every participant completing the seminar with an average above 90%.

Attending the Numerical Linear Algebra for Financial Engineering seminar and passing the final exam meets the linear algebra prerequisites for the Baruch MFE Program. Upon request, recommendation letters reflecting performance in the seminar will also be provided.

**Registration:** To register or to receive more information about the Numerical Linear Algebra for Financial Engineering seminar, send an email to [baruch.mfe@baruch.cuny.edu](mailto:baruch.mfe@baruch.cuny.edu).

### Textbook:

"A Linear Algebra Primer for Financial Engineering", by Dan Stefanica, FE Press, 2014.

**Prerequisites:** Students should read in advance the following sections from the textbook:

Chapter 1, Sections 1.1 - 1.5 (not 1.1.1, 1.2.1, 1.4.1)

Chapter 4, Sections 4.1 - 4.3

Chapter 5, Section 5.1

and do the following exercises:

Chapter 1, Section 1.7: # 1, 2, 4, 6-10

Chapter 4, Section 4.7: # 1-8, 11-13, 15, 16

## Detailed Syllabus

### Session 1:

- Vectors and matrices. Matrix column form and row form. Vector–vector, matrix–vector and matrix–matrix multiplication.
- Transpose of a matrix. Symmetric matrices.
- Matrix rank, nullspace and range of a matrix. Linear independent vectors.
- Non-singular matrices and the inverse of a matrix. The determinant of a matrix.
- Diagonal matrices. Matrix multiplication by diagonal matrices.

*Financial Applications:*

- Covariance matrix computation from time series data.
- Matrix setup for a one period market model example.
- Payoff matrix. Replication of derivative securities. Redundant securities.
- Converting between covariance and correlation matrices.

### Session 2

- Special families of matrices and their properties: lower triangular matrices, upper triangular matrices, banded matrices.
- Solving linear systems corresponding to upper and lower triangular matrices.
- Forward substitution. Backward substitution. Operation count.
- Forward and backward substitution for bidiagonal matrices.
- LU decomposition. Uniqueness of the LU decomposition.
- Pseudocode for the LU decomposition. Operation count.
- Linear system solution using the LU decomposition.

*Financial Applications:*

- Bond pricing and discount factors.
- Finding discount factors using forward and backward substitution.
- Finding discount factors using the LU decomposition.

### Session 3:

- Solving multiple linear systems corresponding to the same matrix.
- Computing the inverse of a non–singular matrix using the LU algorithm.
- LU decomposition without pivoting for tridiagonal matrices.
- The need for pivoting.
- Permutation matrices.
- LU decomposition with row pivoting.
- Identifying non–singular matrices using the LU algorithm.
- Eigenvalues and eigenvectors. Existence and multiplicities.
- The characteristic polynomial of a matrix.
- Linear independence of eigenvectors.

*Financial Applications:*

- Cubic spline interpolation.
- Cubic spline interpolation for interest rate curves.

**Session 4:** The Arrow-Debreu one period market model.

- Redundant securities and replicable securities.
- Complete markets.
- Arbitrage Pricing Theory. Arbitrage-free markets. Portfolio arbitrage.
- Fundamental Theorem of Asset Pricing.
- State Prices. Elementary insurance contracts.
- Risk-neutral probability and risk-neutral pricing.
- Example of a complete market: the one-period binomial model.
- Binomial tree pricers.
- Arbitrage for negative insurance contracts prices.
- Incomplete markets.
- Example of an incomplete market: the one-period trinomial model.
- One period index options market models.

**Session 5:**

- The diagonal form of a matrix. Diagonalization of matrices with a full set of eigenvectors.
- Diagonally dominant matrices. Gershgorin's theorem and its applications.
- Eigenvalues and eigenvectors of tridiagonal symmetric matrices.
- Inner product.
- Orthogonality. Orthogonal vectors and orthogonal matrices.
- Symmetric matrices. Properties: real eigenvalues, full set of orthogonal eigenvectors.
- The diagonal form of a symmetric matrix.
- Symmetric positive definite matrices and symmetric semipositive definite matrices.
- Diagonally dominant symmetric matrices with positive diagonal entries.
- Eigenvalues of symmetric positive definite and symmetric semipositive definite matrices.
- Sylvester's Criterion for positive definite matrices and semipositive definite matrices.
- Summary of positive definiteness criteria for symmetric matrices.

**Session 6:**

- Cholesky decomposition.
- Existence and uniqueness of the Cholesky decomposition for symmetric positive definite matrices.
- Pseudocode and operation count for the Cholesky decomposition.
- Solving multiple linear systems corresponding to the same symmetric positive definite matrix.
- Computing the inverse and the determinant of a symmetric positive definite matrix.
- Identifying symmetric positive definite matrices using the Cholesky decomposition algorithm.
- Cholesky decomposition of tridiagonal symmetric positive definite matrices.
- Optimal linear solvers for tridiagonal symmetric positive definite matrices.
- Covariance and correlation matrices of random variables.
- Converting between covariance and correlation matrices.
- Positive definiteness of covariance and correlation matrices.
- The Linear Transformation Property.
- Necessary and sufficient conditions for covariance and correlation matrices.
- Multivariate normal random variables. Independence and covariance matrix.
- Linear combinations of normal random variables.

*Financial Applications:*

- The efficient implementation of the cubic spline interpolation.
- Efficient cubic spline interpolation for zero rate curves.
- Covariance and correlation matrix estimation from time series data.
- Finding multivariate normal random variables with a given covariance or correlation matrix.
- Finding normal random variables with a given covariance or correlation matrix.

**Session 7:**

- The classical least squares method.
- Linear regression: ordinary least squares for time series data.
- Implied volatility computation from real market data using OLS.
- Efficient portfolio theory. Value-at-Risk and portfolio VaR.
- Rates of return of portfolios.
- Efficient frontier.
- Examples: portfolios made of one risky asset and a risk-free asset and portfolios made of two risky assets.
- Efficient (optimal) portfolios for investments in two risky assets and one risk-free asset.
- Minimum variance portfolios. Maximum return portfolio.
- Tangency portfolio. Setting up efficient portfolios by investing in the tangency (market) portfolio.
- Minimum variance portfolio with no cash position.
- Value-at-Risk (VaR). Advantages and pitfalls of VaR.
- Portfolios with multivariate normal asset returns. Portfolio VaR.

**Session 8:**

- Ordinary least squares for random variables.
- The intuition behind ordinary least squares for time series data.
- Lagrange multipliers for efficient portfolios computations.
- C-VaR. Subadditivity of risk measures.
- Bond hedging and yield correlations.
- Quadratic forms and symmetric positive definite matrices.
- Quadratic Forms. Built-in symmetry. Equivalent forms. Differentials of Quadratic forms.
- Eigenvalue methods.
- Power method. Inverse iteration. Rayleigh Quotient iteration. Convergence properties.
- The QR Algorithm.