

# Advanced Calculus with Financial Engineering Applications

The Pre-MFE Program at Baruch College

November 2 – December 21, 2011

Mathematical and financial concepts that are fundamental for a successful learning experience in financial engineering graduate programs will be presented and explained in detail. Strong emphasis will be placed on fully understanding the material.

Mathematical topics (selected):

- Numerical integration methods
- Lagrange multipliers
- Convergence of Taylor series expansions
- Finite difference approximations
- Stirling's formula
- Polar coordinates transformations
- Newton's method for higher dimensional problems

Financial topics (selected):

- Bond duration and convexity
- Put–Call parity
- Black–Scholes formula
- Numerical estimation of the Greeks
- Implied volatility
- Bootstrapping for finding interest rate curves

## Dates and Times:

Lectures: November 2, 9, 16, 21, 28, 30, and December 7, 14, 6-10pm

**Instructor:** Dan Stefanica, Director, Baruch College Financial Engineering Program

**Tuition:** \$1,450

Attending the seminar on Advanced Calculus with Financial Engineering Applications and passing the final exam meets the calculus prerequisites for the Baruch MFE Program. Upon request, recommendation letters reflecting performance in the seminar will also be provided.

**Registration:** To register or to receive more information about the Advanced Calculus with Financial Applications Seminar, send an email to [baruch.mfe@gmail.com](mailto:baruch.mfe@gmail.com)

## Textbooks:

“A Primer for the Mathematics of Financial Engineering”, Second Edition, by Dan Stefanica, FE Press, 2011.

“Solutions Manual – A Primer for the Mathematics of Financial Engineering”, Second Edition, by Dan Stefanica, FE Press, 2011.

**Prerequisites:** Students should read in advance the following sections from the textbook:  
Chapter 1, Sections 1.1 - 1.10  
Chapter 2, Sections 2.6, 2.7  
Chapter 3, Sections 3.1 - 3.4  
and do the exercises from the first chapter (Section 1.12).

## Detailed Syllabus

### Session 1:

- Brief review of elementary calculus from a more formal point of view: product rule, quotient rule, chain rule.
- Antiderivatives and definite integrals. Fundamental Theorem of Calculus. Integration by Substitution - connection to chain rule. Integration by Parts - connection to Product Rule.
- Differentiating definite integrals with respect to the integral limits and with respect to a parameter under the integral sign.
- Partial Derivatives.

*Financial Applications:*

- European Call and Put options. Payoff and Price diagrams. American Call and Put options.
- The concept of no-arbitrage.
- Put-Call parity.

*Textbook Sections:* Chapter 1.

### Session 2:

- Convergence and evaluation of improper integrals.
- Differentiating improper integrals with respect to the integral limits.
- Numerical methods for approximating definite integrals: the Midpoint, Trapezoidal, and Simpson's rules.
- Convergence of Numerical Algorithms – practical considerations. Approximation errors and the order of convergence of a numerical algorithm.
- The order of convergence of the midpoint, trapezoidal, and Simpson's rule.

*Financial Applications:*

- Interest Rate Curves. Zero Rates. Forward rates and instantaneous forward rates.
- Bond Pricing. Yield of a Bond. Bond Duration. Convexity of a bond.
- Modified Duration and Macaulay Duration.
- Dollar Duration and DV01 for bond portfolio hedging.

*Textbook Sections:* Chapter 2.

### Session 3:

- Discrete probability, mean and variance.
- Random variables. Density function and cumulative distribution. Mean and Variance for random variables.
- The standard normal variable.
- Normal random variables. Mean and Variance.

*Financial Applications:*

- A lognormal model for the evolution of stock prices.
- The Black–Scholes formula.
- Computing the Black–Scholes formula using numerical integration methods.
- Numerical accuracy of the Black–Scholes formula.

*Textbook Sections:* Chapter 3.

**Session 4:**

- Change of density function for functions of random variables.
- Lognormal random variables.
- Connection between the density functions of normal and lognormal variables.
- Chebyshev’s Inequality.

*Financial Applications:*

- Computing the probability that a European Call or Put expires in the money.
- The concept of hedging.  $\Delta$ –hedging and  $\Gamma$ –hedging for options.
- Formulas for the Greeks of plain vanilla European Call and Put options.
- Computing the Greeks of derivative securities without closed formulas: finite difference approximations.

*Textbook Sections:* Chapter 4.

**Session 5:**

- Taylor’s Formula: infinite series and integral residual. Convergence issues.
- Taylor series expansions. Taylor expansions for exponential and logarithmic functions.

*Financial Applications:*

- Risk–neutral pricing and the Black–Scholes formula.
- Interpretation of the  $N(d_1)$  and  $N(d_2)$  terms in the Black–Scholes formula as the Delta of the call option and the risk–neutral probability that the call option expires in the money.
- Approximation of the Black–Scholes formula for at–the–money options.
- Connections between duration, convexity, and the relative change in the value of a bond for parallel shifts in the yield curve.

*Textbook Sections:* Chapter 6.

**Session 6:**

- Finite Difference approximations for first order derivatives: forward, backward, central approximations.
- Finite Difference approximations for higher order derivatives. Order of approximation.
- Example: first order and second order ODEs with constant coefficients on the unit interval. Finite difference discretization and solution.
- Need for fast solvers of linear matrix equations.
- Convergence and order of convergence for finite difference approximations of solutions of ODEs.
- The Black–Scholes PDE.

*Financial Applications:*

- Finite difference approximations for the Greeks.
- Connections between the Greeks using Taylor's formula.
- When do  $\Theta$  and  $\Gamma$  have different signs?
- Barrier Options. Arbitrage arguments for the up-and-in up-and-out parity.

*Textbook Sections:* Chapter 7.

**Session 7:**

- Double integrals. Switching the order of integration (Fubini's Theorem).
- Geometric series.
- Convergence of infinite series. Radius of convergence.
- Stirling's formula.
- Chain rule for functions of several variables.
- Change of Variables for Double Integrals.
- Finding relative extrema for functions of several variables.

*Financial Applications:*

- Show that the density function of the standard normal distribution has unit integral over  $\mathbb{R}$ .
- Forward and Futures contracts.
- Barrier Options.

*Textbook Sections:* Chapter 8.

**Session 8:**

- Lagrange multipliers for finding absolute extrema of multivariable functions.
- Newton's method, bisection method, and secant method for solving nonlinear equations.

*Financial Applications:*

- Finding maximum return and minimum variance portfolios using Lagrange multipliers.
- Lagrange multipliers for Monte Carlo applications.
- Bootstrap method for finding the zero-rate curve.
- Computing implied volatility from the Black-Scholes model.

*Textbook Sections:* Chapters 5 and 9.