Numerical Linear Algebra for Financial Engineering

The Pre-MFE Program at Baruch College

February 1 – March 28, 2012

A numerical view of linear algebra concepts that are fundamental for a successful learning experience in financial engineering graduate programs will be presented in this seminar. Emphasis will be placed on numerical linear algebra methods and their implementation, and on financial applications.

Mathematical topics (selected):

- Direct methods for solving linear systems. Backward and Forward Substitution. LU decomposition.
- Eigenvalues and eigenvectors. Diagonal decomposition.
- Eigenvalue methods. Power Method. QR Algorithm.
- Symmetric matrices. Diagonal Decomposition of symmetric matrices.
- Symmetric positive definite matrices. Diagonal dominance. Gershgorin's theorem.

Financial topics (selected):

- Arrow-Debreu Model.
- Complete and Incomplete Markets - binomial and trinomial trees.
- Covariance and Correlation Matrices.
- Least Squares. Linear Regression.
- Portfolio optimization.

Dates and Times:
Lectures: February 1, 8, 15, 22, 29, and March 7, 14, 21, 6-10pm
Final Exam: Wednesday, March 28, 7-9pm

Instructor: Dan Stefanica, Director, Baruch College Financial Engineering Program

Tuition: $1,450

Attending the Numerical Linear Algebra for Financial Engineering seminar and passing the final exam meets the linear algebra prerequisites for the Baruch MFE Program. Upon request, recommendation letters reflecting performance in the seminar will also be provided.

Registration: To register or to receive more information about the Numerical Linear Algebra for Financial Engineering seminar, send an email to **baruch.mfe@gmail.com**

Textbooks:

• Instructor's Notes

• "Introduction to Linear Algebra", by Gilbert Strang, Wellesley-Cambridge Press, 4th Edition, 2009.

Prerequisites: Upon registration, students will be provided with reading material covering a typical one third of a linear algebra class, and with relevant exercises to be completed before the first day of class.

Detailed Syllabus

Session 1:

- Vectors and matrices. Operations with matrices.
- Column-wise matrix–vector and matrix-matrix multiplication.
- Transpose of a matrix.
- Non-singular matrices and the inverse of a matrix.
- Vector spaces. Linear independence.
- Range and nullspace of a matrix. Rank of a matrix.
- Special families of matrices and their properties: symmetric matrices, diagonal matrices, upper and lower triangular matrices, banded matrices.
- Matrix multiplication by diagonal matrices.

Financial Applications:

- Inferring the relationship between covariance and correlation matrices.
- Bond pricing and discount factors.

Session 2

- Solving linear systems corresponding to upper and lower triangular matrices.
- Forward and Backward Substitution. Operation count.
- LU decomposition. Uniqueness of the LU decomposition.
- Pseudocode for the LU decomposition. Operation count.
- Solving linear systems using the LU decomposition.

Financial Applications:

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- Matrix setup for a one period market model example using bond prices and swap rates.
- Payoff matrix. Redundant securities. Replication of derivative securities.
- Finding discount factors using forward and backward substitution.
- Finding discount factors using the LU decomposition.

Session 3:

- Identifying non–singular matrices using the LU algorithm. Computing the determinant of a matrix using the LU algorithm. Computing the inverse of a non–singular matrix using the LU algorithm.
- LU decomposition for banded matrices.
- The need for pivoting.
- Permutation matrices.
- LU decomposition with row pivoting.
- Eigenvalues and eigenvectors. Existence and multiplicities.
- Eigenvalue decomposition of a matrix.
- Linear independence of eigenvectors.

- Diagonalization of matrices with a full set of eigenvectors.
- Diagonally dominated matrices. Gershgorin's theorem and its applications.

Session 4: The Arrow-Debreu Model for a one period market with m-securities and n-states.

- Redundant securities and replicable securities.
- Complete markets.
- Arbitrage Pricing Theory. Arbitrage-free markets. Portfolio arbitrage. Arbitrage-free markets.
- Fundamental Theorem of Asset Pricing.
- State Prices. Elementary insurance contracts.
- Risk-neutral probability and risk-neutral pricing.
- Arbitrage for negative insurance contracts prices.
- Incomplete Markets.

Session 5:

- Inner product.
- Orthogonality. Orthogonal vectors and orthogonal matrices.
- Diagonal decomposition of a symmetric matrix.
- Special properties of symmetric matrices: real eigenvalues, full set of eigenvectors, diagonal form.
- Symmetric positive definite matrices; properties and equivalent definitions.
- Symmetric semipositive definite matrices.
- Eigenvalue methods.
- Power method. Inverse iteration. Rayleigh Quotient iteration. Convergence properties.
- The QR Algorithm.

Session 6: Efficient Portfolios Theory.
Financial Applications:

- Rates of return of portfolios.
- Efficient frontier.
- Examples: portfolios made of one risky asset and a risk–free asset and portfolios made of two risky assets.
- Efficient (optimal) portfolios for investments in two risky assets and one risk–free asset.
- Minimum variance portfolio. Tangency portfolio.
- Portfolio theory with short positions.
- Multi–asset portfolio theory. Efficient portfolios. Minimum variance portfolios.

Session 7:

- Quadratic forms and symmetric positive definite matrices.
- Quadratic Forms. Built–in symmetry. Equivalent forms. Differentials of Quadratic forms.
- Diagonally dominated matrices.
- Cholesky decomposition.
- Solving linear systems corresponding to symmetric positive definite matrices.

- Least Squares problems.

Financial Applications:

- Covariance matrix. Correlation matrix.
- Linear Transformation Property.
- Finding normal random variables with given correlation matrix.

Session 8:

- Vector norms and matrix norms. Matrix norms induced by vector norms. The 1-norm, 2-norm, and inf-norm.
- Equivalent norms. Equivalence of norms of finite dimensional vector spaces.
- The condition number of a matrix. Well-conditioned and ill-condition matrices and consequences for numerical algorithms.
- The 2-norm and the condition number for symmetric matrices.
- Relevance of condition numbers to solving linear equations.

Financial Applications:

- Transition probability matrices. Credit migration.
- Example of a complete market: the one-period binomial model.
- Binomial tree pricers.
- Example of an incomplete market: the one-period trinomial model.