Fall 2009: MTH 9821 Numerical Linear Algebra

Tuesday 6–8:30pm, Room 10-180

Instructor: Dan Stefanica

Textbooks:

Other Reference Books:

Detailed Syllabus

- **Week 1**
  - Brief review of linear algebra.
  - Direct methods for solving linear problems.
  - Forward and backward substitution.
  - LU decomposition. Definition, existence and uniqueness issues.
  - Pseudocode and operation count for the LU decomposition.
  - Applications of the LU decomposition: computing the determinant of a matrix; solving multiple linear systems corresponding to the same matrix; computing the inverse of a nonsingular matrix.
  - LU decomposition for banded matrices.


- **Week 2**
  - Need for pivoting for the LU decomposition.
  - Permutation matrices: properties and storage.
  - LU decomposition with (partial) row pivoting.
  - LU decomposition with row pivoting for banded matrices.
  - Symmetric positive definite matrices.
  - Cholesky factorization.

Readings: Trefethen, Lectures 21, 23. Instructor’s Notes.

- **Week 3**
  - Existence and uniqueness of the Cholesky decomposition.
  - Cholesky factorization for banded and sparse matrices.
  - Iterative Methods for solving Linear Systems. Example: Richardson iteration.
Readings: Demmel, Chapter 6. Trefethen, Lectures 11, 22, 32. Instructor’s Notes.

• Week 4
  • Jacobi, Gauss-Siedel, and SOR methods. Pseudocodes.
  • Convergence analysis for the Jacobi, Gauss-Siedel, and SOR iterative methods.
  • Comparing the convergence speed of different methods.
  • Least Squares Method.


• Week 5
  • Convergence speed of iterative methods.
  • Eigenvalue problems.
  • Power Method.
  • Inverse Power Method.

Readings: Trefethen, Lectures 24, 25, 28. Instructor’s Notes.

• Week 6 Midterm Exam

• Week 7
  • Equity, Index, and Currency options.
  • The Black-Scholes Formula.
  • Approximation of the BS formula for at the money options.
  • Put-Call parity. No-arbitrage principle.
  • American Call options on non-dividend paying underlying.
  • Valuing plain vanilla options for limiting cases.
  • Greeks. Hedging.

Readings: Wilmott, Chapter 3. Clewlow, Chapter 1. Instructor’s Notes.

• Week 8
  • Derivation of the Black-Scholes PDE. Properties of the Black-Scholes PDE.
  • Financial interpretation of the terms from the Black-Scholes PDE.
  • Change of variables to reduce the BS PDE to the diffusion equation.
  • Boundary conditions for the Black-Scholes PDE and the effect of the change of variables.
  • Closed form solution of the heat equation.
  • Derivation of the Black-Scholes formulas.
  • Derivation of the Black-Scholes PDE for multi-assets options.


• Week 9
  • Finite difference approximations. Finite difference discretization and solution of a second order ODE.
  • Finite difference methods for solving the heat PDE: Forward Euler, Backward Euler, and Crank-Nicolson.
  • Finite difference discretization and solution of the diffusion equation.


• Week 10
  • Finite difference methods for solving the BS PDE. Boundary Conditions.
  • Pricing European plain vanilla options using Forward Euler, Backward Euler, and Crank-Nicolson.
  • Comparison of domain discretizations for solving the Black-Scholes PDE.
• Finite difference approximations of the Greeks.

Readings: Wilmott, Chapters 4, 8. Clewlow, Chapter 3. Instructor’s Notes.

• Week 11
  • Projected SOR.
  • Forward and Backward Euler schemes for pricing American plain vanilla options.

Readings: Wilmott, Chapters 8, 9. Clewlow, Chapter 3. Instructor’s Notes.

• Week 12
  • Barrier options. Closed Formulas. Arbitrage pricing.
  • Pricing European barrier options using Forward Euler, Backward Euler, and Crank-Nicolson.
  • Domain discretization for pricing Bermudan options using finite difference methods.
  • Forward Euler, Backward Euler, and Crank-Nicolson for Bermudan options pricing and Greeks approximations.

Readings: Wilmott, Chapter 12.

• Week 13
  • Domain discretization for finite difference pricing of options on underlying assets paying discrete dividends.
  • Finite difference approximations of option values and Greeks for plain vanilla, barrier, and Bermudan options on underlying assets paying discrete dividends.
  • Implied volatility computations using finite difference methods.

• Week 14
  • Derivation of the Barone-Adesi–Whaley approximate formula for American plain vanilla options.
  • Newton’s method and the implementation of the Barone-Adesi–Whaley formula.
  • Computing an approximate implied volatility for American plain vanilla options using the Barone-Adesi–Whaley formula.